1972

Estimation of concrete strains and prestress losses in pretensioned members (Preliminary Draft), May 1972

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Prestress Losses in Pretensioned

Concrete Structural Members

ESTIMATION OF CONCRETE STRAINS AND PRESTRESS LOSSES IN PRETENSIONED MEMBERS

by

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Fritz Engineering Laboratory Report No. 339.7

Lehigh University

Research Project 339 Reports

PRESTRESS LOSSES IN PRETENSIONED CONCRETE STRUCTURAL MEMBERS

COMPARATIVE STUDY OF SEVERAL CONCRETES REGARDING THEIR POTENTIALS FOR CONTRIBUTING TO PRESTRESS LOSSES.
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ESTIMATION OF CONCRETE STRAINS AND PRESTRESS LOSSES IN PRETENSIONED MEMBERS. Ying, H.-T., Schultchen, E. and Huang, T., F. L. Report 339.7, May 1972

COMMONWEALTH OF PENNSYLVANIA

Department of Transportation

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Project 66-17: Prestress Losses in Pretensioned Concrete Structural Members

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and
PRESTRESS LOSSES IN PRETENSIONED MEMBERS

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Bethlehem, Pennsylvania

May, 1972

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TABLE OF CONTENTS

		Page
	ABSTRACT	vi
1.	INTRODUCTION	1
2.	DATA REDUCTION AND ANALYSIS	3
	2.1 Method of Analysis	3
	2.2 Data Reduction	3
	2.3 Two-Dimensional Analysis	4
	2.4 Three-Dimensional Analysis	6
3.	COMPONENTS OF CONCRETE STRAINS	9
	3.1 Components of Concrete Strains	9
	3.2 Elastic Shortening	9
	3.3 Shrinkage Strain	10
	3.4 Creep Strain	11
	3.5 Estimation of Concrete Stress, f_{c3}	13
4.	COMBINED CONCRETE AND STEEL STRESS-STRAIN-TIME RELATIONSHIP	15
	4.1 Introduction	15
	4.2 Steel Functions	15
	4.3 Concrete Functions	16
	4.4 Steel-Concrete Relationships	17
5.	DISCUSSION OF RESULTS AND SUMMARY	22
	5.1 Application of Prediction Method	22
	5.2 Comparison of Predicted Results (PRELOA) and Three-Dimensional Results (CUNIFB)	23
	5.3 Effects of Types of Concrete and Initial Prestress	23

		Page
	5.4 Summary	24
6.	ACKNOWLEDGMENTS	25
7.	TABLES	26
8.	FIGURES	39
9.	REFERENCES	45
	APPENDIX - NOTATIONS	46

LIST OF TABLES

Table		Page
1	Two-Dimensional Results for Uniformly Stressed Specimens	27
2	Two-Dimensional Results for Shrinkage Specimens	28
3	Three-Dimensional Results for Uniformly Stressed Specimens	29
4	Three-Dimensional Results for Shrinkage Specimens	29
5	Coefficients of Shrinkage Function	30
6	Predicted Shrinkage Strain at 100 Years	30
7	Coefficients of Creep Function	31
8	Predicted Creep Strain at 100 Years	31
9	Comparison of f_{s3} for CUNIFC and PRELOA	32
10	Values of k_1, k_2, k_3 and μ for Various Series of Specimens	33
11	Prestress Losses Predicted by PRELOA	34
12	Predicted Concrete Strains at 10 Days	35
13	Predicted Concrete Strains at 100 Days	36
14	Predicted Concrete Strains at 1000 Days	37
15	Predicted Concrete Strains at 100 Years	38

LIST OF FIGURES

Figure		Page
1	Stress Distributions for Rectangular Specimens	40
2	Time-Dependent Concrete Strain vs. Time for Plant AB	41
3	Time-Dependent Concrete Strain vs. Time for Plant CD	42
4	Shrinkage Strain vs. Time for Plant AB	43
5	Shrinkage Strain vs. Time for Plant CD	44

ABSTRACT

This report describes a long-term study of concrete strains in pretensioned concrete structural members. Test data over a period of three years have been included in this study. The concrete strains are separated into three components: elastic shortening, shrinkage and creep and a functional expression is developed for each. Combining these components, a general stress-strain-time relationship of concrete is derived which allows the estimation of the concrete strain when the other two variables are given.

The concrete stress-strain-time relationship is further combined with a similar relationship for the prestressing strands, and a general analytical procedure for the prediction of prestress loss is developed. Application of this general procedure to uniformly (centrally) prestressed concrete members is demonstrated, and results compared with the experimentally obtained values. Pending further verifications, this same procedure, with minor modifications, is expected to be also applicable to members prestressed non-uniformly (eccentricity) and/or subjected to external loading.

1. INTRODUCTION

The research project "Prestress Losses in Pretensioned Concrete Structural Members" was initiated in 1966 with the objective of establishing a rational basis for the prediction of prestress losses in pretensioned highway bridge members used in Pennsylvania. In two separate phases of the project, concrete and steel specimens are being tested for the several major factors contributing to prestress losses. To date, several reports have been issued describing the experimental work, and presenting preliminary results and conclusions.

Prestress losses due to elastic shortening, shrinkage and creep in concrete were investigated by testing concrete specimens. From a preliminary study of the loss characteristics of several concretes (Ref. 1), two plants had been selected to produce these specimens, one with concrete exhibiting the highest loss characteristics (plant AB) and the other the lowest (plant CD). A total of twenty-two prestressed specimens were fabricated at each plant. By controlling the number and location of prestressing strands, variations of average prestress level and lateral stress gradient were obtained, as shown in Fig. 1. In addition, twenty unprestressed shrinkage specimens were also fabricated at each plant. The majority of these specimens contain unstretched strands corresponding to the various prestressed specimens. There is also a series of shrinkage specimens with no strands in them. A detailed description of the test specimens and experiment was given in Ref. 2.

The preliminary results, at the end of one year, of this phase of the experiment have been presented in Ref. 2. This present report contains the final results based on data from the shrinkage and the uniformly stressed specimens. Test data over a period of three years have been included in the analysis.

In Ref. 3 a concept of stress-strain-time relationship was introduced and applied to the steel relaxation data. This same concept will be applied in this report to the concrete strain data. The concrete and steel stress-strain-time relationships will then be combined to form a basis for the prediction of prestress losses. Numerical results of prestress losses predicted by this method will be shown, and compared with the experimental data.

In Chapter 2, measured concrete strains from the shrinkage and uniformly prestressed specimens are analyzed two- and three-dimensionally to determine the most suitable time and stress function forms. Chapter 3 contains the analysis of the separate components of concrete strain, using the function form selected in Chapter 2. The development of the complete concrete stress-strain-time relationship and of the general analysis procedure are presented in Chapter 4.

2. DATA REDUCTION AND ANALYSIS

2.1 Method of Analysis

The procedure of analysis used on the concrete strain data is similar to that which was adopted in the development of the steel stress-strain-time relationship (Ref. 3). First of all, a two-dimensional analysis was performed. The concrete strain data for each given initial concrete stress were analyzed as a function of time only. After the selection of the time function, the data from all specimens were then analyzed "three-dimensionally", treating both the initial concrete stress and the time as independent variables. In this chapter, the initial concrete stress refers to the nominal concrete stress obtained by dividing the total prestress force by the gross sectional area of the member.

2.2 Data Reduction

Both shrinkage and uniformly stressed specimens data were reduced in the same manner. The readings taken immediately after the transfer of prestressing force were used as the bases of reference. To determine the concrete strains at subsequent times, the corresponding readings were subtracted from these initial readings. The differences represented respectively the change of gage length due to shrinkage, in the case of shrinkage specimens, and that due to the combination of shrinkage, creep and relaxation, in the case of prestressed specimens. By using the after release readings, rather than the before release readings, as the bases of reference, the effect of elastic shortening was excluded.

All data used in both two-dimensional and three-dimensional analysis were reduced in the same manner as described in the preceding paragraph.

2.3 Two-Dimensional Analysis

The time-dependent natures of concrete strain and steel relaxation are quite similar. The same "essential conditions" listed in Ref. 3 should also be satisfied by the time function of concrete losses. Therefore, the same test functions used in the two-dimensional analysis of steel relaxation were also used in the analysis of concrete strain data:

1. $S(t) = A_1 + A_2 \log t + A_3 (\log t)^2$
2. $S(t) = A_1 + A_2 \log t$
3. $S(t) = A_1 + A_2 \log (t + 1) + A_3 [\log (t + 1)]^2$
4. $S(t) = A_1 + A_2 \log (t + 1)$

A computer program, CUNIFA, was developed to perform the two-dimensional regression analysis, using the method of least squares. Strain data from a single series of specimens (consisting of two concrete specimens, and up to 40 strain readings at any given time) were used, and time after transfer of prestress was the only independent variable.

The same criteria used in Ref. 3 for selecting the best time function were used. The functions were evaluated primarily on the basis of their stability, that is, whether their long-term predictions and behavior were strongly affected by the inclusion and exclusion of data near either end of the test period. Standard errors were also compared to see how well each test function approximated the data.

Functions (1) and (3) were affected strongly by experimental data near the beginning of the test period, and therefore were rejected. Both functions (2) and (4) were relatively insensitive to the variations of the test period, although the standard errors were slightly higher than those of functions (1) and (3) because of their simpler forms. Between these two, function (4) showed less sensitivity to changes of the beginning time of the test period and also resulted in lower standard errors. Therefore, function (4) was considered to be the best time function, and was selected to be used in the three-dimensional analysis. Incidentally, function (4) is the same time function which was selected for relaxation of steel.

Values of predicted strains at 100 years, and standard errors for the different series of uniformly stressed and shrinkage specimens are listed in Table 1 and 2 respectively.

2.4 Three-Dimensional Analysis

In the two-dimensional analysis, the data analysis was restricted to time-dependent functions which allowed the prediction of concrete strains for single sets of data corresponding to a particular level of initial concrete stress. A logical extension of this concept would be to include the initial nominal stress as a second independent variable besides time.

As in Ref. 3 the stress-dependent trial functions were obtained from linear combinations of 1, f_i, f_i^2, where f_i was the initial concrete stress.

1. $S(f_i) = B_1 + B_2 f_i + B_3 f_i^2$

2. $S(f_i) = B_1 + B_2 f_i$

3. $S(f_i) = B_1 + B_2 f_i^2$

4. $S(f_i) + B_1 f_i + B_2 f_i^2$

In a fairly general form the concrete strains as a function of initial stress and time can be expressed as

$$S(f_i, t) = \sum_{m=1}^{M} \sum_{n=1}^{N} a_{mn} F_m(f_i) T_n(t)$$

where $F_m(f_i)$ are subfunctions of the initial stress f_i only and $T_n(t)$ are subfunctions of time t. With the time-dependent function already selected by the two-dimensional analysis, only four functions were tested in the three-dimensional analysis:

1. $S(f_i,t) = C_1 + C_2 \log(t + 1) + f_i [C_3 + C_4 \log(t + 1)]$
$+ f_i^2 [C_5 + C_6 \log(t + 1)]$

2. $S(f_i,t) = C_1 + C_2 \log(t + 1) + f_i [C_3 + C_4 \log(t + 1)]$

3. $S(f_i,t) = C_1 + C_2 \log(t + 1) + f_i^2 [C_3 + C_4 \log(t + 1)]$

4. $S(f_i,t) = f_i [C_1 + C_2 \log(t + 1)] + f_i^2 [C_3 + C_4 \log(t + 1)]$

Strain data were regressed with respect to time and initial nominal stress. This was done by a computer program, CUNIFB, again using the method of least square. The test functions were evaluated by the same criteria used in the three-dimensional analysis of steel relaxation in Ref. 3.

Function (1) has the lowest standard errors due to its larger number of regression coefficients. However, the magnitudes and signs for these coefficients differed greatly for the concrete specimens from the two different plants. This indicated a redundancy in the number of regression coefficients in the function. Accordingly, function (1) was rejected. Function (4) had the highest standard errors, hence was judged not suitable for prediction purposes. Both functions (2) and (3) behaved rather well. Function (2) was preferred because of its simpler form and slightly lower standard errors.

With the selected function (2), three regression analyses were performed, one for each plant separately and the third for both plants combined. The standard errors together with the predicted strains at 100 years for different initial concrete stresses for the uniformly stressed and shrinkage specimens are listed in Table 3 and 4 respectively. The functions for the uniformly stressed and shrinkage specimens from plant AB and plant CD are also plotted in Figs. 2 - 5.

3. COMPONENTS OF CONCRETE STRAINS

3.1 Components of Concrete Strains

In a prestressed concrete member, the concrete strains contributing to the prestress loss can be separated into three components: elastic shortening, shrinkage and creep. In this chapter, the effect of each was studied separately and a prediction formula developed for the respective components. By combining the three components the total concrete strain can be predicted.

The variables and units used in the development of all the prediction formulas were:

S: concrete strain, in 0.01 in./in. (elastic, shrinkage and creep components designated by subscripts el, sh and cr, respectively)

f_c: concrete stress, in ksi

f_s: steel stress, in ksi

t: time after transfer of prestress, in days

3.2 Elastic Shortening

In the prediction of elastic shortening, the idealized linear elastic stress-strain relationship of concrete was utilized.

$$S_{el}(f_c) = C_1 f_c \tag{3-1}$$

where $C_1 = 100/E_c$

E_c = elastic modulus of concrete at time of transfer, in ksi

Standard cylinder tests were used to determine the secant modulus of the concretes studied in this project. The average values for plant AB, plant CD, and their average were 4000 ksi, 4750 ksi, and 4350 ksi, respectively.

It should be pointed out that the concrete stress, f_c, in equation (3-1) refers to that at any arbitrary time, t, after the transfer of prestress. As such, the equation includes the effect of elastic rebound of concrete due to the gradual decrease of concrete stress.

3.3 Shrinkage Strain

As described in Chapter 2, one of the independent variables for the three-dimensional analysis was the "nominal concrete stress", obtained by dividing the total tensioning force in strands by the gross sectional area of the specimen. For the shrinkage data, the nominal stress in the companion uniformly stressed specimens was used in the analysis. However, as these shrinkage specimens were actually not prestressed, a more logical choice of parameter would be the amount of reinforcement, reflecting the restraining effect of steel on the shrinkage of concrete. Therefore, in the final analysis of shrinkage data, the parameter "nominal concrete stress" was replaced by the amount of reinforcement (including both prestressing strands and non-prestressing bars). The revised three-dimensional function takes the following form,

$$S_{sh}(\mu,t) = D_1 + D_2 \log(t+1) + \mu\,[D_3 + D_4 \log(t+1)] \qquad (3\text{-}2)$$

where μ = amount of reinforcement, in percent of the gross cross section area

t = time after end of curing, taken as same as time after transfer of prestress, in days

D_1, D_2, D_3, D_4 = regression coefficients

The shrinkage coefficients in equation (3-2) were determined separately for plant AB concrete, plant CD concrete and for the two plants combined, and are listed in Table 5. The predicted shrinkage strains at 100 years for various steel percentages are listed in Table 6.

3.4 Creep Strain

The reduced strain data used in the three-dimensional analysis of the uniformly stressed specimens included shrinkage and creep of concrete as well as the elastic rebound of concrete strain due to decreasing concrete stress. In order to isolate the effect of creep, strains due to shrinkage and elastic rebound must be deducted from the experimental values for total strain.

Shrinkage strains were calculated by equation (3-2). Elastic rebound was calculated by using the linear elastic stress-strain relationship

$$S_{er} = C_1 (f_{c3} - f_c) \tag{3-3}$$

where S_{er} = elastic rebound strain, in 0.01 in/in

f_{c3} = concrete stress at transfer of prestress, in ksi

The creep strain of concrete S_{cr}, was then calculated as

$$S_{cr} = S_{ct} - S_{sh} + S_{er} \tag{3-4}$$

where S_{ct} = total strain of uniformly stressed specimens

For the estimation of the elastic rebound, and for the regression analysis of creep strain data, concrete stress f_c at each data time must first be evaluated. This was accomplished by first calculating the stress in steel strands by the steel stress-strain-time relationship with strain and time known. The concrete stress was then calculated from the equilibrium conditions.

The concrete stress immediately after transfer of prestress, f_{c3}, was also required in equation (3-3). It was estimated by the method described in Section 3.5.

A computer program, CUNIFC, was developed to perform the necessary calculations and the regression analysis of the isolated creep strain data. The regression function used was the same one selected in the three-dimensional analysis.

$$S_{cr}(f_c,t) = E_1 + E_2 \log(t + 1) + f_c [E_3 + E_4 \log(t + 1)] \qquad (3\text{-}5)$$

where E_1, E_2, E_3, E_4 are regression coefficients

The creep coefficients in equation (3-5) were determined separately for plant AB concrete, plant CD concrete and for the two combined, and are listed in Table 7. The predicted strains at 100 years for different constant concrete stresses are listed in Table 8.

3.5 Estimation of Concrete Stress, f_{c3}

For the concrete specimens used in this research project, no direct measurement of steel stress (or strain) was possible after detensioning of the prestressing bed. The stress conditions immediately after transfer must therefore be estimated indirectly. In computer program CUNIFC, the after-transfer stresses were estimated in the following manner. As a first step, the steel stress-strain-time relationship (Ref. 3) was used to evaluate the relaxation loss for the time period from tensioning to immediately before transfer. This calculated value, however, did not reflect the effect of elevated temperature which existed during this period of time. It is known that relaxation increases rather substantially under continuous high temperature. On the other hand, experimental data are not currently available for a quantitative determination of the short-duration temperature effect during the curing period. An increase of 30% was assumed, resulting in an estimated total relaxation loss before transfer of from 4.2 to 4.7% of the initial steel stress for plant AB specimens and from 3.8 to 4.3% for plant CD specimens. (The average lengths of curing period were three days and two days for the two plants, respectively.) To simplify calculations, an average value of 4.5% was used for plant AB and 4.0% for plant CD. Subtracting these values from the initial tensioning stress resulted in the steel stress immediately before transfer, f_{s2}.

The difference between the concrete strain readings taken immediately before and after release of prestressing force represented the elastic strain. Multiplying this elastic strain by the elastic modulus of steel, taken as 28000 ksi, resulted in the elastic loss of steel stress. Subtracting this elastic loss from the steel stress immediately before transfer, f_{s2}, the steel stress after tranfer, f_{s3}, was obtained. The corresponding after-release concrete stress, f_{c3}, was simply calculated from the equilibrium condition between the steel and concrete stresses.

It will be noted later that in the general procedure for prediction of prestress losses, a different procedure is used for the estimation of after-transfer stress condition. Further comments will be given in Section 4.4.

4. COMBINED CONCRETE AND STEEL STRESS-STRAIN-TIME RELATIONSHIP

4.1 Introduction

In this chapter, the functions separately developed for steel (Ref. 3) and concrete (Chapter 3) are integrated to yield the combined form of "concrete-steel" stress-strain-time relationship. The steel and concrete functions are recollected in Sections 4.2 and 4.3. Section 4.4 describes the linkage of the steel and concrete stress-strain-time relationships, and outlines the procedure of the application of the combined relationship.

4.2 Steel Functions

The following steel functions were developed in Ref. 3.

(1) Stress-strain relationship

$$f_{s_{el}}(S_s) = \left\{ A_1 + A_2 S_s + A_3 S_s^2 \right\} f_{pu} \tag{4-1}$$

(2) Relaxation loss of steel stress

$$L(S_s, t_s) = \Big\{ S_s \left[B_1 + B_2 \log (t_s + 1) \right] + S_s^2 \left[B_3 + B_4 \log (t_s + 1) \right] \Big\} f_{pu} \tag{4-2}$$

(3) Stress-strain-time relationship

$$\begin{aligned} f_s(S_s, t_s) &= f_{s_{el}}(S_s) - L(S_s, t_s) \\ &= \Big\{ A_1 + S_s \left[A_2 - B_1 - B_2 \log (t_s + 1) \right] \\ &\quad + S_s^2 \left[A_3 - B_3 - B_4 \log (t_s + 1) \right] \Big\} f_{pu} \end{aligned} \tag{4-3}$$

where S_s = steel strain, in 0.01 in/in

t_s = time after stretching of strands, in days

L = relaxation loss, in ksi

f_s = steel stress, in ksi

f_{pu} = guaranteed ultimate strength of steel, in ksi

A_1, A_2, A_3 = regression coefficients of stress-strain curve of steel

B_1, B_2, B_3, B_4 = regression coefficients of relaxation of steel

The applicability of the stress-strain-time relationship equation (4-3) is restricted to time values from approximately one day to 100 years and to initial stress values in the range from 0.5 to 0.8 of the ultimate tensile strength.

4.3 Concrete Functions

(1) Elastic shortening

$$S_{el}(f_c) = C_1 f_c \tag{4-4}$$

(2) Shrinkage

$$S_{sh}(\mu, t_c) = D_1 + D_2 \log (t_c + 1) + \mu \left[D_3 + D_4 \log (t_c + 1)\right] \tag{4-5}$$

(3) Creep

$$S_{cr}(f_c,t_c) = E_1 + E_2 \log (t_c + 1) + f_c[E_3 + E_4 \log (t_c + 1)] \quad (4\text{-}6)$$

(4) Total concrete strain

$$S_c(\mu,f_c,t_c) = S_{el} + S_{sh} + S_{cr}$$

$$= [D_1 + E_1 + \mu D_3 + (D_2 + E_2 + \mu D_4) \log (t_c + 1)]$$

$$+ [C_1 + E_3 + E_4 \log (t_c + 1)] f_c \quad (4\text{-}7)$$

Equations (4-4), (4-5) and (4-6) are identical to equations (3-1), (3-2) and (3-5), respectively, which were developed in Chapter 3. All parameters have been defined previously, except that the time parameter is now denoted by t_c. The total concrete strain was obtained by simply combining the individual concrete components. The range of applicability of equation (4-7) is, in accordance to the data source, for t_c to be from approximately one day to 100 years, and for f_c to be below 3 ksi.

4.4 Steel-Concrete Relationships

Combining the stress-strain-time relationships of steel and concrete materials, a general procedure for the prediction of stress conditions in a prestressed concrete member can be established. Initially, the development of this general procedure was based on the uniformly (centrally) prestressed specimens. For these specimens, the stress, strain and time parameters for the two materials are related by equations (4-8), (4-9) and (4-10)

$$S_s = k_1 - S_c \qquad (4\text{-}8)$$

$$f_c = k_2 f_s \qquad (4\text{-}9)$$

$$t_s = k_3 + t_c \qquad (4\text{-}10)$$

where k_1 = steel strain at initial stretching, in 0.01 in./in.

k_2 = A_{ps}/A_c, dimensionless

A_{ps} = Area of prestressing steel, in sq. in.

A_c = net concrete cross-sectional area, in sq. in. (equal to the gross area subtracting area of prestressing steel)

k_3 = time from tensioning of steel to transfer of stress, in days

Equation (4-8) is the geometric compatibility condition for the steel and concrete strains, equation (4-9) is the equilibrium condition between the steel and concrete forces at a cross-section, and equation (4-10) merely shows the difference in the definitions of the time parameters.

Substituting (4-10) into (4-3),

$$f_s = x_1 + x_2 S_s + x_3 S_s^2 \qquad (4\text{-}11)$$

where $x_1 = A_1 f_{pu}$

$$x_2 = \left\{A_2 - B_1 - B_2 \log (t_c + k_3 + 1)\right\} f_{pu}$$

$$x_3 = \left\{A_3 - B_3 - B_4 \log (t_c + k_3 + 1)\right\} f_{pu} \qquad (4\text{-}11a)$$

Substituting (4-8) into (4-11)

$$f_s = (x_1 + x_2k_1 + x_3k_1^2) - (x_2 + 2x_3k_1) S_c + x_3S_c^2 \qquad (4\text{-}12)$$

Rewriting (4-7)

$$S_c = y_1 + y_2f_c \qquad (4\text{-}13)$$

where $y_1 = D_1 + E_1 + \mu D_3 + (D_2 + E_2 + \mu D_4) \log (t_c + 1)$

$$y_2 = C_1 + E_3 + E_4 \log (t_c + 1) \qquad (4\text{-}13a)$$

Substituting (4-9) into (4-13)

$$S_c = y_1 + y_2k_2f_s \qquad (4\text{-}14)$$

Substituting (4-14) into (4-12)

$$f_x = [(x_1 + x_2k_1 + x_3k_1^2) - (x_2 + 2x_3k_1) y_1 + x_3y_1^2]$$

$$- [(x_2 + 2x_3k_1 - 2x_3y_1) y_2k_2f_s] + [x_3y_2^2k_2^2f_s^2] \qquad (4\text{-}15)$$

Equation (4-15) is recognized as a quadratic equation for f_s

$$Z_1 + Z_2f_s + Z_3f_s^2 = 0 \qquad (4\text{-}16)$$

$$\text{where } Z_1 = (x_1 + x_2k_1 + x_3k_1^2) - (x_2 + 2x_3k_1)\ y_1 + x_3y_1^2$$

$$Z_2 = -[(x_2 + 2x_3k_1 - 2x_3y_1)\ y_2k_2 + 1]$$

$$Z_3 = x_3y_2^2k_2^2 \tag{4-16a}$$

For a given type of concrete and prestressing strands, the parameters k_1, k_2 and k_3 completely define the prestressed concrete member. Once these three values have been given, the steel stress, f_s, can be calculated for any given time, t_c, by equation (4-16). The corresponding concrete stress, concrete strain and steel strain can be obtained from equation (4-9), equation (4-13) and equation (4-8) respectively. A computer program PRELOA was developed to perform all the calculations involved in the prediction of prestress losses as outlined above.

It has been mentioned earlier that the procedure for estimating the after-transfer stress conditions used in developing the creep expressions (Section 3.5) was not used in the prediction procedure. In procedure PRELOA, no provision was made for the short duration temperature effect on pre-transfer relaxation loss, as any such provision would violate the linking conditions. The stress condition immediately after transfer was solved from equation (4-16) by assigning $t_c = 0$. In addition, creep and shrinkage strains were eliminated by setting $y_1 = 0$ and $y_2 = C_1$. The steel stress immediately after transfer, f_{s3}, predicted by the procedure PRELOA was slightly higher than that estimated by CUNIFC. Table 9 exhibits a comparison for the ten series of uniformly prestressed specimens. It is observed that the two sets of f_{s_3} values differ by only 2 to 3.5%, which is indeed tolerable.

It should be pointed out that equation (4-16) is applicable only to uniformly (centrally) prestressed members subjected to no external loading. For members nonuniformly (eccentrically) prestressed, and/or subjected to external loading, the equilibrium condition (4-9) would have to be generalized. However, the same concept of combining the two stress-strain-time relationships should be equally valid. Therefore, subject to further modifications and verifications, this general analytical procedure is expected to be applicable to any pretensioned prestressed concrete member.

5. DISCUSSION OF RESULTS AND SUMMARY

5.1 Application of Prediction Method

As an initial test of feasibility of the prediction method described in Chapter 4, the procedure was applied to the ten series of uniformly stressed specimens. Based on the known initial stress condition, geometrical design and the fabrication schedule (k_1, k_2 and k_3), predicted prestress losses and concrete strain at various times were calculated. Values of k_1, k_2 and k_3 as well as those of μ, the amount of steel reinforcement, for the ten series of uniformly stressed specimens are shown in Table 10.

The predicted prestress losses at various times for the different series of specimens are listed in Table 11. The total prestress losses at 100 years, taken as the lifetime of a bridge structure, varied from 33% to 47% of the initial prestress for plant AB concrete and 24% to 37% for plant CD concrete. It should be pointed out, however, that these high values of prestress loss were due to the fact that the specimens were not externally loaded. For actual structural members, externally applied dead and live loads would reduce the concrete stress in the members considerably, and lower percentage losses of prestress would be typical.

5.2 Comparison of Predicted Results (PRELOA) with Three-Dimensional Results (CUNIFB)

The predicted concrete strains were compared with the results of the earlier three-dimensional analysis (CUNIFB). The predicted concrete strains and the comparisons, in terms of percentage differences, at times 10, 100, 1000 and 36500 days are shown in Tables 12, 13, 14 and 15 respectively.

In examining the tables, it should be borne in mind that the entire analysis has been based on test data of the first 1000 days. The values in Table 15, for 100 years, were therefore obtained through extrapolation and their accuracy cannot be verified.

The comparisons showed that the results from the three-dimensional analysis and the predicted results agree very well. The maximum deviation is 12%. However, in most cases, the deviations are about 5% or less.

5.3 Effects of Types of Concrete and Initial Prestress

In Table 11 are shown prestress losses at various times expressed as a fraction of the 100 year loss (taken as the ultimate value). It can be observed that for the same initial prestress level, the two concrete mixes showed approximately the same prestress losses at any given time. In other words, the type of concrete does not have any significant effect on the growth of prestress loss. On the other hand, concrete from plant

CD consistently showed lower prestress losses (absolute) than concrete from plant AB. This was expected since concrete from plant CD had lower loss characteristics than that from plant AB (as was mentioned in Chapter 1).

It can also be clearly observed that higher initial prestresses yield higher prestress losses.

5.4 Summary

Two stress-strain-time relationships were developed separately for steel and concrete. These two relationships were then integrated to form a combined "concrete-steel" stress-strain-time relationship, which was applied on the different series of test specimens to predict concrete strains and prestress losses. Comparison of these predicted results and the results from the three-dimensional analysis showed that the developed prediction method applied very well.

It was observed that the growth of prestress loss did not depend on the type of concrete used, and that the higher the initial prestress level the higher the prestress loss.

The validity of the proposed prediction method in this report is yet to be justified by comparisons with the test data of non-uniformly stressed specimens. This is logically the next step to be taken in this project.

6. ACKNOWLEDGMENTS

The study reported herein is a part of a research project at the Fritz Engineering Laboratory, Lehigh University, under the auspices of the Office of Research. The project is financially sponsored by the Pennsylvania Department of Transportation, the U. S. Department of Transportation, Federal Highway Administration; and the Reinforced Concrete Research Council.

The help rendered by Mr. John Gera in tracing the graphs, and that by Mrs. Ruth Grimes in typing the manuscript, are gratefully acknowledged.

7. TABLES

TABLE 1: TWO-DIMENSIONAL RESULTS FOR UNIFORMLY STRESSED SPECIMENS

Plant	Series	Predicted* Strain	Standard Error
AB	1.0	.1700	.0067
	1.5	.1635	.0083
	2.0	.2132	.0059
	3.0	.2398	.0071
	3.6	.2440	.0083
CD	1.0	.0917	.0037
	1.5	.1156	.0047
	2.0	.1159	.0050
	3.0	.1545	.0060
	3.6	.1801	.0053

* Predicted strains at t = 36500 Days

TABLE 2: TWO-DIMENSIONAL RESULTS FOR SHRINKAGE SPECIMENS

Plant	Series	Predicted* Strain	Standard Error
AB	0.0+	.1011	.0059
	1.0	.0851	.0055
	1.5	.0909	.0052
	2.0	.0937	.0051
	3.0	.0683	.0048
	3.6	.0641	.0041
CD	0.0+	.0558	.0041
	1.0	.0589	.0046
	1.5	.0602	.0037
	2.0	.0397	.0033
	3.0	.0362	.0042
	3.6	.0430	.0034

* Predicted strains at t = 36500 days

\+ Shrinkage specimens in these series contain no strand

TABLE 3: THREE-DIMENSIONAL RESULTS FOR UNIFORMLY STRESSED SPECIMENS

Plant	Predicted Strain After 100 Years					Standard Error
	1.0*	1.5*	2.0*	3.0*	3.6*	
AB	.1630	.1798	.1966	.2301	.2502	.0084
CD	.0928	.1087	.1245	.1562	.1752	.0057
AB-CD	.1274	.1450	.1626	.1979	.2190	.0190

* Initial Nominal Concrete Stress in ksi, same as series name

TABLE 4: THREE-DIMENSIONAL RESULTS FOR SHRINKAGE SPECIMENS

Plant	Predicted Strain After 100 Years						Standard Error
	0.0*	1.0*	1.5*	2.0*	3.0*	3.6*	
AB	.0995	.0914	.0873	.0832	.0750	.0701	.0059
CD	.0677	.0585	.0539	.0494	.0402	.0347	.0050
AB-CD	.0857	.0765	.0720	.0674	.0583	.0528	.0096

* Initial Nominal Concrete Stress in ksi, same as series name

TABLE 5: COEFFICIENTS OF SHRINKAGE FUNCTION

$$\text{Shrinkage Strain} = D_1 + D_2 \log(t+1) + \mu\,[D_3 + D_4 \log(t+1)]$$

Plant	Shrinkage Coefficients			
	D_1	D_2	D_3	D_4
AB	-.0042	.0230	.0033	-.0040
CD	-.0021	.0159	.0012	-.0042
AB-CD	-.0025	.0198	.0014	-.0041

TABLE 6: PREDICTED SHRINKAGE STRAIN AT 100 YEARS

Steel Percentage, μ*	Plant		
	AB	CD	AB-CD
0.0	.1007	.0706	.0877
0.5	.0933	.0615	.0791
1.0	.0859	.0525	.0705
1.5	.0785	.0435	.0618
2.0	.0711	.0344	.0532
2.5	.0637	.0254	.0446
3.0	.0563	.0164	.0360
3.5	.0488	.0074	.0274

* For correspondence to Series, see Table 10

TABLE 7 COEFFICIENTS OF CREEP FUNCTION

$$\text{Creep Strain} = E_1 + E_2 \log (t + 1) + f_c [E_3 + E_4 \log (t + 1)]$$

Plant	Creep Coefficients			
	E_1	E_2	E_3	E_4
AB	-.0161	.0087	-.0018	.0185
CD	-.0082	-.0023	-.0030	.0161
AB-CD	-.0165	.0074	.0022	.0140

TABLE 8: PREDICTED CREEP STRAIN AT 100 YEARS

Constant Concrete Stress ksi	Plant		
	AB	CD	AB-CD
0.5	.0649	.0167	.0503
1.0	.1063	.0520	.0834
1.5	.1476	.0873	.1165
2.0	.1890	.1226	.1496
2.5	.2303	.1579	.1827
3.0	.2717	.1932	.2158
3.5	.3131	.2285	.2488

TABLE 9: COMPARISON OF f_{s3} FOR CUNIFC* AND PRELOA+

Series		$\frac{f_{s1}}{f_{pu}}$	f_{s3}/f_{pu}	
			CUNIFC	PRELOA
Plant AB	1.0	.6645	.6103	.6224
	1.5	.6968	.6293	.6414
	2.0	.6645	.5834	.6025
	3.0	.6968	.5954	.6130
	3.6	.6968	.5835	.6021
Plant CD	1.0	.6645	.6212	.6273
	1.5	.6968	.6384	.6480
	2.0	.6645	.5981	.6105
	3.0	.6968	.6087	.6236
	3.6	.6968	.5922	.6141

* CUNIFC - Creep Analysis (Section 3.4)
(allowing temp. effect on pre-transfer relaxation)

\+ PRELOA - Prediction Method (Section 4.4)
(ignoring temp. effect on pre-transfer relaxation)

TABLE 10: VALUES OF k_1, k_2, k_3 AND μ

FOR VARIOUS SERIES OF SPECIMENS

Plant	Series	k_1 (.01 in/in)	k_2	k_3 (days)	μ (%)
AB	1.0	.6393	0.00563	3.17	.65
	1.5	.6721	0.00804	3.10	.89
	2.0	.6393	0.01130	3.10	1.21
	3.0	.6721	0.01622	3.04	1.70
	3.6	.6721	0.01952	3.00	2.03
CD	1.0	.6393	0.00563	2.19	.65
	1.5	.6721	0.00804	2.17	.89
	2.0	.6393	0.01130	2.00	1.21
	3.0	.6721	0.01622	1.96	1.70
	3.6	.6721	0.01952	1.92	2.03

TABLE 11: PRESTRESS LOSSES PREDICTED BY PRELOA

Series		$\frac{100(f_{s1} - f_{s100})}{f_{s1}}$	$100(f_{s3} - f_s)/(f_{s3} - f_{s100})$			
			10 Days	100 Days	1000 Days	10000 Days
Plant AB	1.0	33.3	17.3	42.5	66.5	88.5
	1.5	35.9	18.9	44.0	67.7	88.9
	2.0	39.8	20.4	45.5	68.7	89.3
	3.0	44.1	22.6	47.8	70.3	90.0
	3.6	46.9	23.9	49.0	71.3	90.4
Plant CD	1.0	24.3	17.1	42.3	66.5	88.5
	1.5	27.1	18.6	43.8	67.6	88.9
	2.0	30.0	19.8	45.1	68.5	89.3
	3.0	34.5	21.5	47.0	70.0	89.9
	3.6	37.2	22.5	48.1	70.7	90.2

Note: f_{s1} = Initial Steel Stress at Stretching

f_{s100} = Steel Stress at 100 Years

TABLE 12: PREDICTED CONCRETE STRAINS AT 10 DAYS

Series		Concrete Strain		$\frac{100\,(\text{CUNIFB}-\text{PRELOA})}{\text{CUNIFB}}$
		CUNIFB*	PRELOA+	
Plant AB	1.0	.0304	.0266	12.5
	1.5	.0346	.0329	4.9
	2.0	.0389	.0385	1.0
	3.0	.0475	.0488	2.7
	3.6	.0526	.0541	2.9
Plant CD	1.0	.0149	.0138	7.4
	1.5	.0185	.0184	0.5
	2.0	.0220	.0226	3.0
	3.0	.0291	.0305	4.8
	3.6	.0334	.0347	3.9

* CUNIFB - Results of Three-Dimensional Analysis

\+ PRELOA - Predicted Results

TABLE 13: PREDICTED CONCRETE STRAINS AT 100 DAYS

Series		Concrete Strain		$\frac{100(\text{CUNIFB}-\text{PRELOA})}{\text{CUNIFB}}$
		CUNIFB*	PRELOA+	
Plant AB	1.0	.0667	.0665	0.3
	1.5	.0743	.0770	3.6
	2.0	.0820	.0856	4.8
	3.0	.0974	.1019	4.6
	3.6	.1066	.1097	2.9
Plant CD	1.0	.0362	.0362	0.0
	1.5	.0432	.0451	4.4
	2.0	.0501	.0527	5.2
	3.0	.0639	.0673	5.3
	3.6	.0722	.0747	3.5

* CUNIFB - Results of Three-Dimensional Analysis

\+ PRELOA - Predicted Results

TABLE 14: PREDICTED CONCRETE STRAINS AT 1000 DAYS

Series		Concrete Strain		$\frac{100\,(\text{CUNIFB}-\text{PRELOA})}{\text{CUNIFB}}$
		CUNIFB*	PRELOA+	
Plant AB	1.0	.1042	.1057	1.4
	1.5	.1154	.1195	3.5
	2.0	.1266	.1298	2.5
	3.0	.1491	.1502	0.7
	3.6	.1625	.1593	2.0
Plant CD	1.0	.0583	.0580	0.5
	1.5	.0687	.0706	2.8
	2.0	.0791	.0810	2.4
	3.0	.0998	.1009	1.1
	3.6	.1123	.1106	1.5

* CUNIFB - Results of Three-Dimensional Analysis

\+ PRELOA - Predicted Results

TABLE 15: PREDICTED CONCRETE STRAINS AT 100 YEARS

Series		Concrete Strain		$\frac{100(\text{CUNIFB}-\text{PRELOA})}{\text{CUNIFB}}$
		CUNIFB*	PRELOA+	
Plant AB	1.0	.1630	.1635	0.3
	1.5	.1798	.1808	0.6
	2.0	.1966	.1919	2.1
	3.0	.2301	.2156	6.3
	3.6	.2502	.2249	10.1
Plant CD	1.0	.0928	.0901	2.9
	1.5	.1087	.1073	1.3
	2.0	.1245	.1209	2.9
	3.0	.1562	.1467	6.1
	3.6	.1752	.1586	9.5

* CUNIFB - Results of Three-Dimensional Analysis

\+ PRELOA - Predicted Results

8. FIGURES

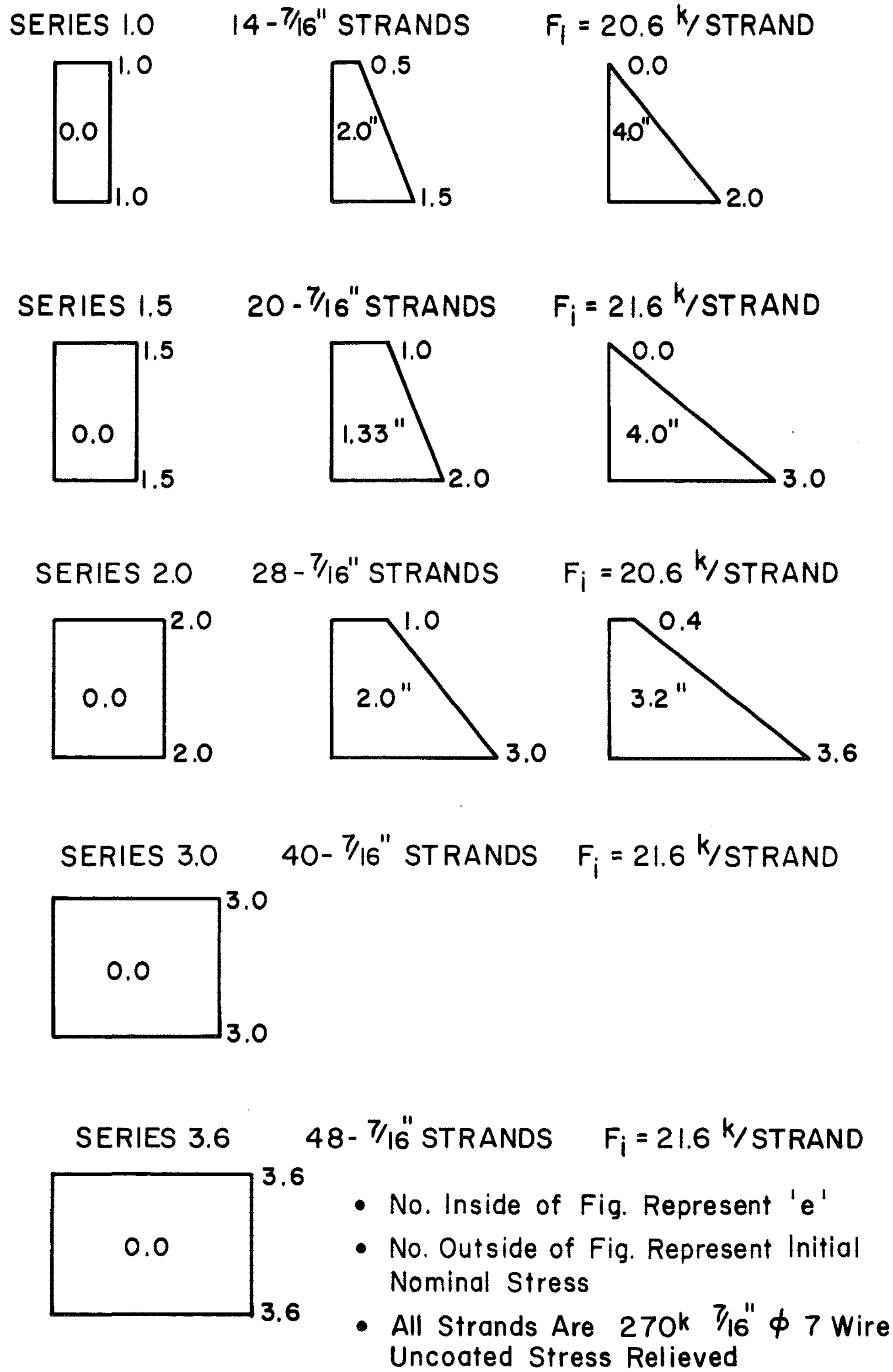

Fig. 1 Stress Distributions for Rectangular Specimens

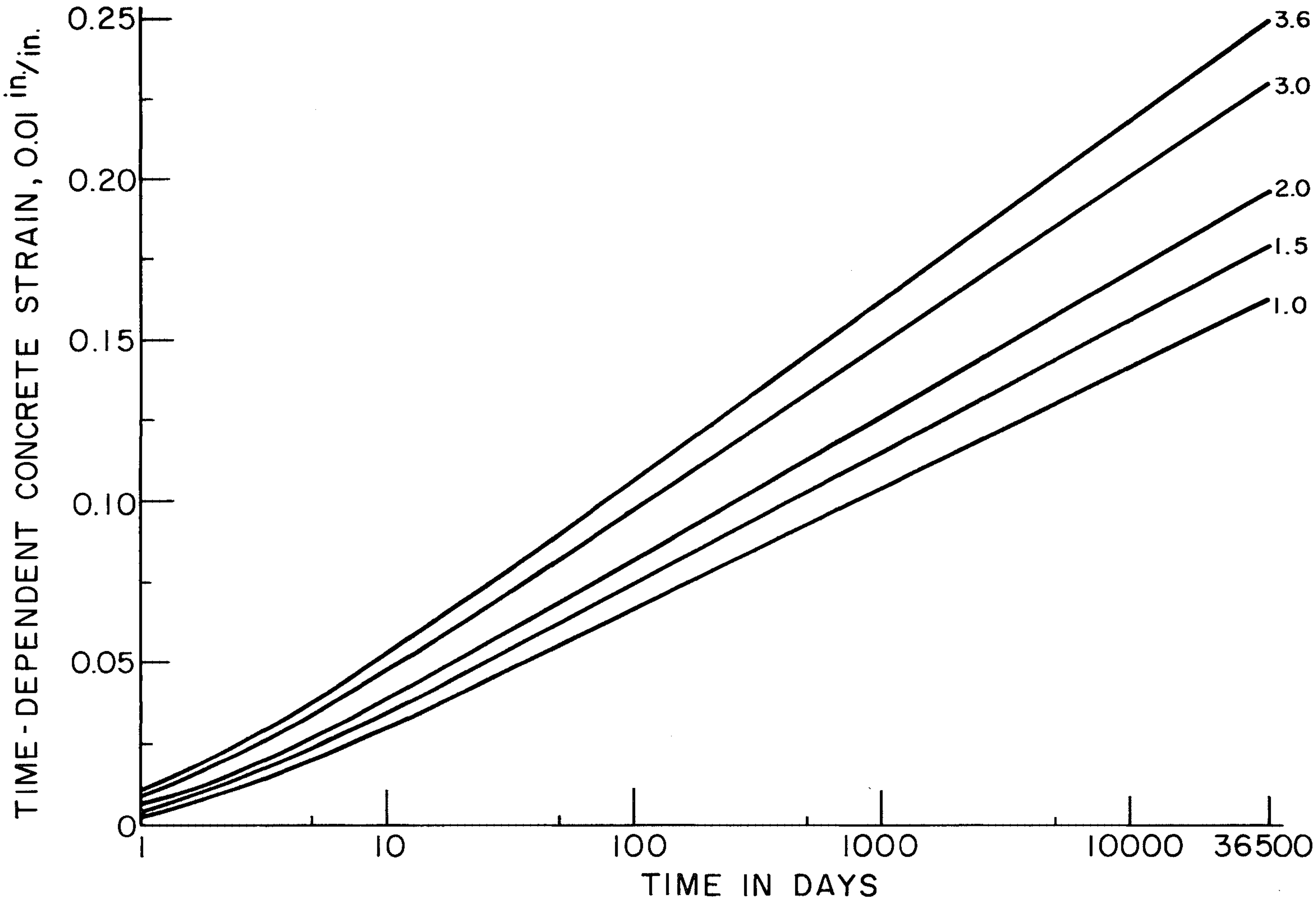

Fig. 2 Time-Dependent Concrete Strain vs. Time for Plant AB

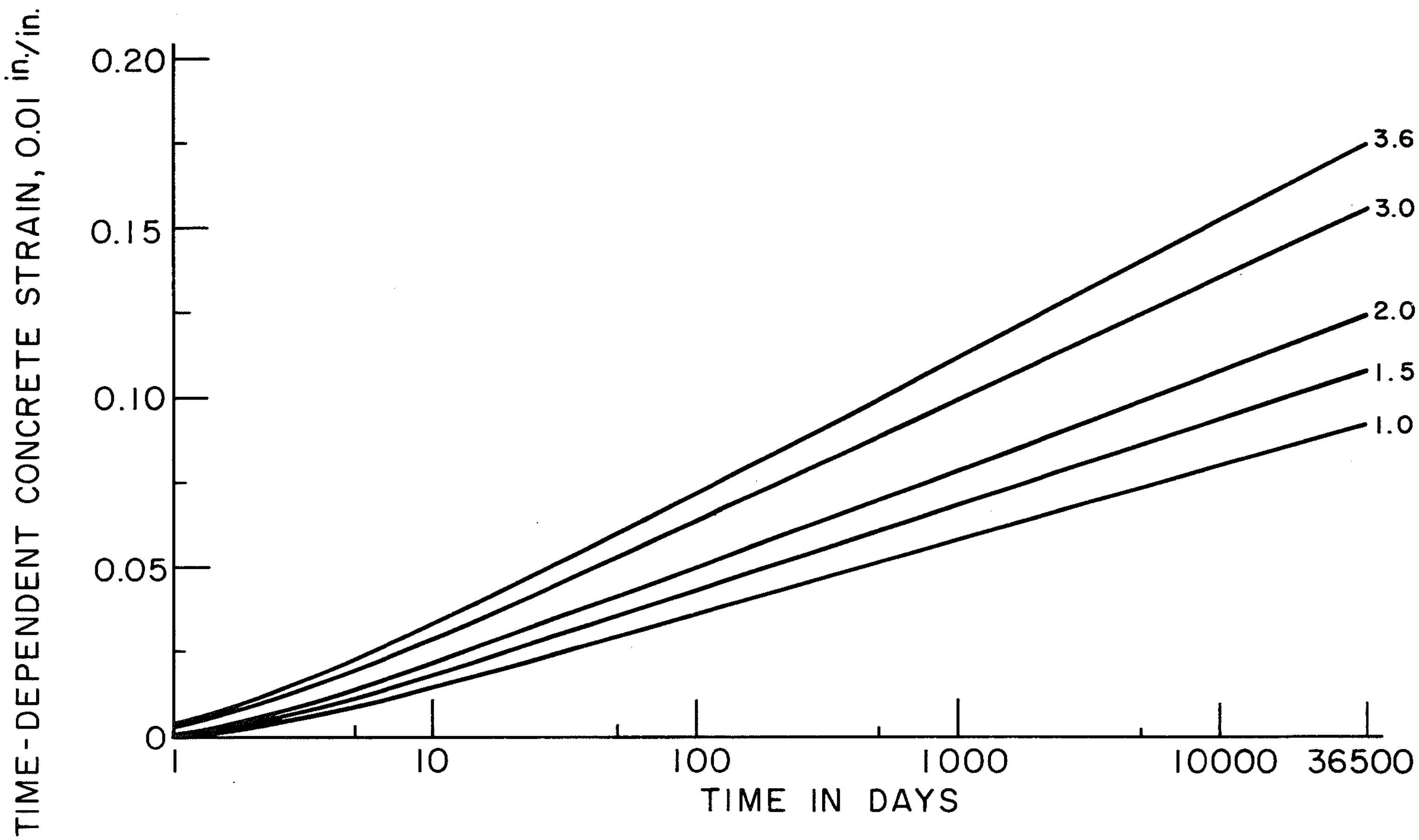

Fig. 3 Time-Dependent Concrete Strain vs. Time for Plant CD

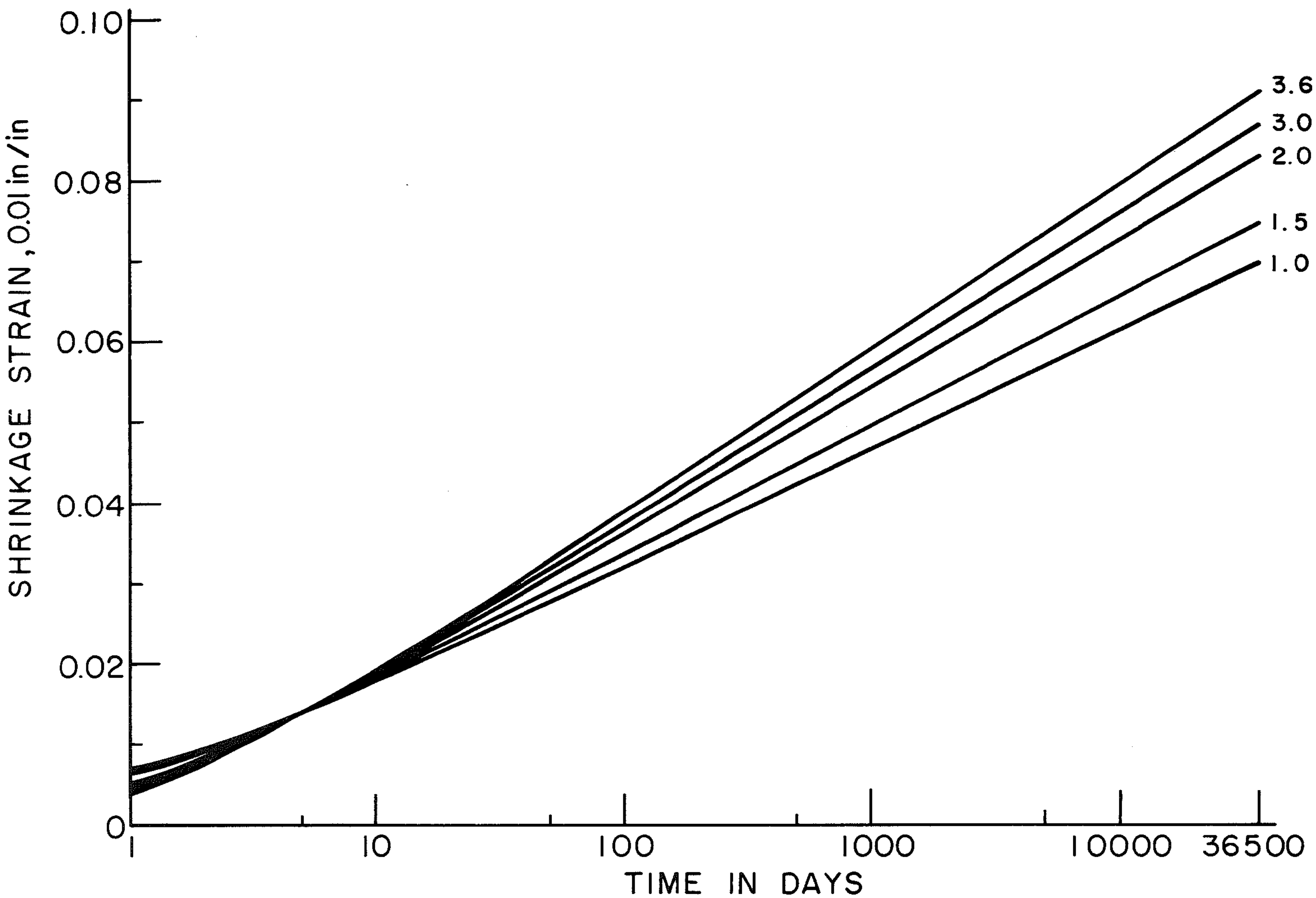

Fig. 4 Shrinkage Strain vs. Time for Plant AB

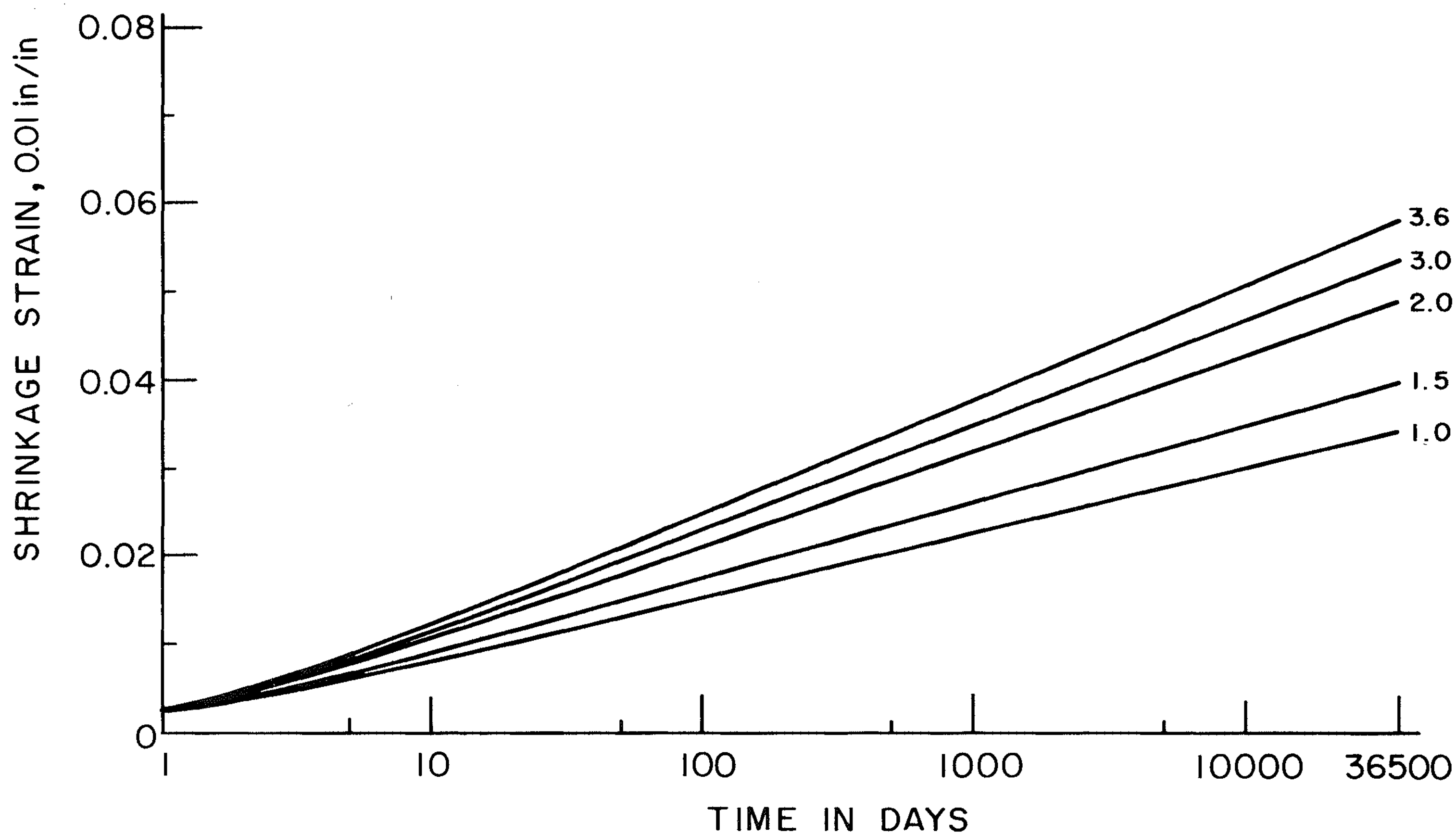

Fig. 5 Shrinkage Strain vs. Time for Plant CD

9. REFERENCES

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APPENDIX

NOTATIONS

All notations have been defined when they first appeared in this report. They are assembled here for easy reference. Unless specifically indicated otherwise, all quantities are in consistent inch-kip-day units. All strain quantities are in 0.01 in./in.

A_c = Area of net concrete cross section (gross section area subtract the area of prestressing steel)

A_{ps} = Area of prestressing steel

E_c = Secant modulus of elasticity of concrete

f_c = Stress in concrete

f_i = Initial concrete nominal stress (used in CUNIFC)

f_{pu} = Ultimate tensile strength of steel

f_s = Stress in steel

k_1 = Initial strain of steel at tensioning, in 0.01 in./in.

k_2 = Ratio of A_{ps} to A_c, dimensionless

k_3 = Time interval from initial tensioning to transfer

L = Loss of steel stress due to relaxation

S_c = Concrete strain

S_{cr} = Creep strain of concrete

S_{ct} = Total measured concrete strain

S_{el} = Elastic strain of concrete

S_{er} = Elastic rebound strain of concrete

S_s = Steel strain

S_{sh} = Shrinkage strain of concrete

t_c = "Concrete time", starting from transfer

t_s = "Steel time", starting from tensioning

μ = Amount of longitudinal steel, both prestressed and non-prestressed, in percent of gross section area

Numerical subscripts used with stress notations to designate time:

1 Time of initial tensional

2 Immediately before transfer

3 Immediately after transfer